## On – Line Programme of Learning for Year 13 – Further Mathematics

| Autumn  |   | Spring   |   | Summer  |          |
|---|---|--|---|---|----------|
| From: September   | To: December  | From: January  | To: April   | From: May   | To: July |
| Teacher 1:Topic – A2 Pure MathsIntegrationTopic: - Core Pure 2SeriesMethods in CalculusMethods in DifferentialEquationsTopic – Further Pure 1VectorsInequalitiesThe t-formulae  | Teacher 2:<br>Topic: - Core Pure 2<br>Complex Numbers<br>Volumes of Revolution<br>Polar Coordinates<br>Hyperbolic Functions   | Teacher 1:<br>Topic: - Core Pure 2<br>Modelling with Differential<br>Equations<br>Topic – Further Pure 1<br>Conic Sections<br>Taylor Series<br>Methods in Calculus<br>Numerical Methods<br>Reducible Differential<br>Equations   | Teacher 2:<br>Topic – Further Mechanics 1<br>Momentum and Impulse<br>Elastic Strings and Springs<br>Elastic Collisions in two-<br>dimensions  | From: May To: July Revision for exams External exams will be sat this term. |          |
| Key Skills – A2 Pure Maths<br>Integrate standard<br>mathematical functions<br>including trigonometric and<br>exponential functions and<br>use the reverse of the chain<br>rule to integrate functions of<br>the form f(ax + b)<br>Use trigonometric identities<br>in integration<br>Use the reverse of the chain<br>rule to integrate more<br>complex functions<br>Integrate functions by making<br>a substitution, using<br>integration by parts and using<br>partial fractions<br>Differentiate and integrate<br>parametric equations<br>Key Skills – CP2<br>Understand and use the | Key Skills – CP2<br>Express a complex number in<br>exponential form.<br>Multiply and divide complex<br>numbers in exponential form.<br>Understand de Moivres's<br>theorem.<br>Use de Moivres's theorem to<br>derive trigonometric<br>identities; find sums of series;<br>and to find the nth root of<br>equations.<br>Use complex roots of unity to<br>solve geometric problems.<br>Find volumes of revolution<br>around both the x- and y-axis<br>and for curves defined<br>parametrically.<br>Model real-life applications of<br>volumes of revolution.<br>Understand and use polar | Key Skills – CP2<br>Model real-life situations<br>with first-order differential<br>equations.<br>Use differential equations to<br>model simple harmonic<br>motion.<br>Model damped and forced<br>oscillations using differential<br>equations.<br>Model real-life situations<br>using coupled first-order<br>differential equations.<br>Key Skills – FP1<br>Identify an ellipse or a<br>hyperbola from its Cartesian<br>or parametric equations. Find<br>the foci, directrices, and<br>eccentricity for an ellipse or a<br>hyperbola.<br>Find tangents and normal to | Key Skills – FM1<br>Use the impulse-momentum<br>principle and the principle of<br>conservation of momentum<br>in vector form.<br>Use Hooke's law to solve<br>equilibrium problems<br>involving elastic strings or<br>springs.<br>Use Hooke's law to solve<br>dynamics problems involving<br>elastic strings or springs.<br>Find the energy stored in an<br>elastic string or spring.<br>Solve problems involving<br>elastic energy using the<br>principle of conservation of<br>mechanical energy and the<br>work-energy principle.<br>Solve problems involving the<br>oblique impact of a smooth |   |          |
| finite series<br>Find and use higher<br>derivatives of functions.   | Convert between polar and<br>Cartesian coordinates<br>Sketch polar curves.  | Solve simple loci questions.<br>Derive and use Taylor series<br>for simple functions.  | Solve problems involving the<br>oblique impact of two<br>smooth spheres.  |   |          |

| Know how to express             | Find the area enclosed by a     | Use series expansions to       | Solve problems involving      |  |
|---------------------------------|---------------------------------|--------------------------------|-------------------------------|--|
| functions as an infinite series | polar curve.                    | evaluate limits. Use the       | successive oblique impacts of |  |
| in ascending powers using       | Find tangents parallel to, or   | Taylor series method to find a | a sphere with smooth plane    |  |
| Maclaurin series expansion      | at right angles to, the initial | series solution to a           | surfaces.                     |  |
| Bea bale to find the series     | line.                           | differential equation.         |                               |  |
| expansions of compound          | Understand the definitions of   | Apply Leibnitz's theorem for   |                               |  |
| functions.                      | hyperbolic functions            | differentiating products       |                               |  |
| Evaluate improper integrals     | Sketch the graphs of            | Understand the use of          |                               |  |
| Understand and evaluate the     | hyperbolic functions.           | derivatives to evaluate limits |                               |  |
| mean value of a function.       | Understand and use the          | of indeterminate forms using   |                               |  |
| Integrate rational functions    | inverse hyperbolic functions.   | l'Hospital's rule.             |                               |  |
| using trigonometric             | Prove identities and solve      | Use tangent half-angle         |                               |  |
| substitutions                   | equations using hyperbolic      | substitutions to find definite |                               |  |
| Integrate using partial         | functions.                      | and indefinite integrals using |                               |  |
| fractions.                      | Differentiate and integrate     | Weierstrass substitution.      |                               |  |
| Solve first-order differential  | hyperbolic functions.           | Find numerical solutions to    |                               |  |
| equations using an              | Polar coordinates and           | first-order differential       |                               |  |
| integrating factor.             | hyperbolic functions            | equations using Euler's        |                               |  |
| Solve second-order              | introduce new functions and     | method and the mid-point       |                               |  |
| homogeneous differential        | graphs – linking to previous    | method.                        |                               |  |
| equations using the auxiliary   | skills in coordinate geometry   | Extend Euler's method to find  |                               |  |
| equation.                       | and trigonometry.               | numerical solutions to         |                               |  |
| Solve second-order non-         |                                 | second-order differential      |                               |  |
| homogeneous differential        |                                 | equations.                     |                               |  |
| equations using the             |                                 | Use Simpson's rule to find an  |                               |  |
| complimentary function and      |                                 | approximation for a given      |                               |  |
| the particular integral.        |                                 | definite integral.             |                               |  |
| Find particular solutions to    |                                 | Use a substitution to          |                               |  |
| differential equations using    |                                 | transform first and second-    |                               |  |
| given boundary conditions.      |                                 | order differential equations   |                               |  |
| Key Skills – FP1                |                                 | into ones that can be solved.  |                               |  |
| Write the vector equation of    |                                 | Solve modelling problems       |                               |  |
| a line using the cross          |                                 | involving reducible            |                               |  |
| product.                        |                                 | differential equations.        |                               |  |
| Find the direction rations and  |                                 |                                |                               |  |
| direction cosines of a line.    |                                 |                                |                               |  |
| Use vectors in problems         |                                 |                                |                               |  |
| involving points, lines and     |                                 |                                |                               |  |
| planes                          |                                 |                                |                               |  |
| Use the equivalent Cartesian    |                                 |                                |                               |  |
| forms for the equations of      |                                 |                                |                               |  |
| lines and planes.               |                                 |                                |                               |  |
| Solve inequalities involving    |                                 |                                |                               |  |
| modulus signs                   |                                 |                                |                               |  |

| Use the t-formulae for             |                                     |                                       |                                     |   |                              |
|------------------------------------|-------------------------------------|---------------------------------------|-------------------------------------|---|------------------------------|
| modelling with trigonometry        |                                     |                                       |                                     |   |                              |
| modelling with trigonometry.       |                                     |                                       |                                     |   |                              |
|                                    |                                     |                                       |                                     |   |                              |
|                                    |                                     |                                       |                                     |   |                              |
| Assessments:                       | 1                                   |                                       | 1                                   | 1                                       | I                            |
| At AS students will be assessed    | on their ability to                 |                                       |                                     |   |                              |
| A01 (50%): Use and apply stand     | dard techniques – i.e. be able to   | (i) select and carry out routine pr   | ocedures; (ii) accurately recall fa | cts, terminology and definitions        |                              |
| A02 (at least 15%): Reason, inte   | erpret and communicate mather       | natically – i.e. be able to (i) const | ruct rigorous mathematical argui    | ments (including proofs); (ii) mak      | e deductions and inferences; |
| (iii) assess the validity or mathe | ematical arguments; (iv) explain t  | heir reasoning; (v) use mathematic    | tical language and notation corre   | ectly                                   |                              |
| A03 (at least 15%): Solve proble   | ems within mathematics and in c     | other contexts – i.e. be able to (i)  | translate problems in mathemati     | ical and non-mathematical conte         | xts into mathematical        |
| processes; (ii) interpret solution | ns to problems in their original co | ontext, and, where appropriate, e     | evaluate their accuracy and limita  | ations; (iii) translate situations in ( | context into mathematical    |
| models; (iv) use mathematical      | models; (v) evaluate the outcom     | e of modelling in context. Recogr     | nise the limitations of models and  | l, where appropriate, explain how       | v to refine them.            |
| End of term 1 assessment to co     | over:                               | Term 2 assessment to cover:           |                                     |   |                              |
| All AS content will be assessed    | in September                        | January exams will assess AS co       | ontent for FP! And FM1 and all      |   |                              |
| All A2 content taught in Autum     | n 1 will be assessed in             | Core Pure content covered in y        | ear 12 and the autumn term.         |   |                              |
| November.                          |                                     | ,                                     |                                     |   |                              |
|                                    |                                     |                                       |                                     |   |                              |
| Building understanding:            | Building understanding:             | Building understanding:               | Building understanding:             |   |                              |
| Rationale / breakdown for          | Rationale / breakdown for           | Rationale / breakdown for             | Rationale / breakdown for           |   |                              |
| your sequence of lessons:          | your sequence of lessons:           | your sequence of lessons:             | your sequence of lessons:           |   |                              |
| The key topics in Year 2           | Complex numbers builds on           | Conic Sections builds on the          | This term takes students            |   |                              |
| Mathematics have to be             | the skills learnt at AS on this     | AS content and utilises               | through the final topics in         |   |                              |
| complete before any                | topic. Volumes of revolution        | hyperbolic functions taught           | further mechanics – building        |   |                              |
| progress can be made in Core       | looks at more complex               | last term.                            | on skills learnt at AS.             |   |                              |
| Pure. The topic of series          | models and utilises                 | The remainder of the term             | The mechanics teacher is            |   |                              |
| builds on the CP1 topic and        | integration skills covered in       | takes a sequential approach           | likely to finish teaching early     |   |                              |
| on fluency in year 2               | A2 mathematics.                     | through differential                  | in the term and will move           |   |                              |
| differentiation. Methods in        |                                     | equations building on the             | onto revision of key topics.        |   |                              |
| Calculus relies on fluency in      |                                     | foundations laid last term.           |                                     |   |                              |
| year 2 integration.                |                                     | Some further methods in               |                                     |   |                              |
| Differential equations are         |                                     | calculus are introduced to            |                                     |   |                              |
| introduced this term –             |                                     | extend the calculus skills and        |                                     |   |                              |
| building on A2 skills and          |                                     | we cover numerical methods.           |                                     |   |                              |
| laying the foundation for the      |                                     | The topics covered rely               |                                     |   |                              |
| rest of the course.                |                                     | heavily on fluency in all             |                                     |   |                              |
| Moving on to FP1 topics then       |                                     | algebra and calculus skills           |                                     |   |                              |
| gives time for further practice    |                                     | covered on the course to              |                                     |   |                              |
| and consolidation of calculus      |                                     | date.                                 |                                     |   |                              |
| skills before tacking              |                                     |                                       |                                     |   |                              |
| differential equations.            |                                     |                                       |                                     |   |                              |
| For FP1, the topics covered        |                                     |                                       |                                     |   |                              |
| this term all extend the           |                                     |                                       |                                     |   |                              |
| equivalent AS topics and have      |                                     |                                       |                                     |   |                              |
| no reliance on calculus skills.    |                                     |                                       |                                     |   |                              |

| Whilst students haven't yet  |                                      |   |                                    |                        |  |  |  |
|--|--------------------------------------|---|------------------------------------|------------------------|--|--|--|
| covered modulus function in  |                                      |   |                                    |                        |  |  |  |
| A2 mathematics, the further  |                                      |   |                                    |                        |  |  |  |
| mathematicians have enough   |                                      |   |                                    |                        |  |  |  |
| knowledge of graphs for this   |                                      |   |                                    |                        |  |  |  |
| to not be an issue within  |                                      |   |                                    |                        |  |  |  |
| inequalities.  |                                      |   |                                    |                        |  |  |  |
| Calendared Centrally Planned Extended Home – Learning Tasks:   |                                      |   |                                    |                        |  |  |  |
| End of chapter assessments will take place regularly in lessons  |                                      |   |                                    |                        |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |
| Reading / literacy / Oracy:  | - Ale - Balla and a sub-             |   |                                    |                        |  |  |  |
| For reading in mathematics, see  | e the links under enrichment.        |   |                                    |                        |  |  |  |
| For literacy, students will learn  | how to break down long worded        | problems to extract the mathem            | natics involved. This will be mode | lied in the classroom. |  |  |  |
| Students should get used to rea  | ading all parts of the textbook / ex | xam questions and challenging w           | fords they don't understand.       |                        |  |  |  |
| Numeracy:  |                                      |   |                                    |                        |  |  |  |
| Students should be numerate in terms of knowing what a sensible answer looks like for any question they answer and not simply relying on the calculator.                                 |                                      |   |                                    |                        |  |  |  |
| Enrichment / opportunities to develop cultural capital (including careers, WRL and SMSC):  |                                      |   |                                    |                        |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |
| Plus Magazine https://plus.maths.org/content/- an online magazine that provides articles and podcasts for all aspects of mathematics, often discussing aspects of mathematics underlying |                                      |   |                                    |                        |  |  |  |
| recent news stories. They regularly interview people in maths-based careers and so this is a great source of inspiration for real world mathematics.                                     |                                      |   |                                    |                        |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |
| Imperial College run an on-line programme (including masterclasses and MOOCs) in the spring and summer term for students in year 12 who are considering mathematics at university and    |                                      |   |                                    |                        |  |  |  |
| aiming for an A* - https://www.imperial.ac.uk/be-inspired/schools-outreach/secondary-schools/mentoring-and-tutoring/maths-online-programme/  |                                      |   |                                    |                        |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |
| The practice materials for university admissions provide enrichment and challenge on AS topics for students aspiring to the top grades. TMUA is the most accessible.                     |                                      |   |                                    |                        |  |  |  |
| https://www.admissionstesting.org/for-test-takers/test-of-mathematics-for-university-admission/preparation/; whilst the Oxford MAT papers are more challenging                           |                                      |   |                                    |                        |  |  |  |
| https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test/mat-past-papers  |                                      |   |                                    |                        |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |
| A padlet of resources for enrich   | iment and revision for Feathersto    | one students is kept here <u>https://</u> | /padlet.com/lemerson3/KS5math      | <u>s</u>               |  |  |  |
|  |                                      |   |                                    |                        |  |  |  |